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Critical coupling in strong QED with weak gauge dependence

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Abstract

We study chiral symmetry breaking in quenched QED₄, using a vertex ansatz recently proposed by Curtis and Pennington. Bifurcation analysis is employed in a general covariant gauge to investigate the gauge-dependence of the critical coupling for chiral symmetry breakdown. This turns out to be relatively minor, justifying the use of this vertex.

1. Introduction

Three years ago, one of us introduced an ansatz for the full vertex function of quenched quantum electrodynamics (QED) [1]. This not only ensures satisfaction of the Ward-Takahashi identity and avoids singularities that would imply the existence of a scalar, massless particle, but it also respects the requirement of multiplicative renormalizability, a property of exact QED that is destroyed by the popular ladder or rainbow approximation. It agrees moreover with perturbative results in the weak coupling limit.

In this paper we consider the Dyson-Schwinger equations in a general covariant gauge, with the Curtis-Pennington ansatz, and apply bifurcation analysis to them. This involves calculating the Fréchet derivative of the nonlinear mapping of the mass function into itself. Thanks to the scale-invariance of the problem, the bifurcation equation can be solved by inspection, in the limit that the ultra-violet cut-off is taken to infinity. A solution for the mass-function is a power of the momentum that has to satisfy a certain transcendental equation. The onset of criticality is her-

alded by the coming together of two solutions of this transcendental equation, for that is the indication that oscillatory takes over from non-oscillatory behaviour. This study has been performed independently by the authors in two groups (AGR and BP). Obtaining common results, we have merged to present this work.

We find the gauge dependence of the critical coupling to be slight, varying by only a few percent over a relatively large range of the gauge parameter. This confirms the previous wholly numerical findings of Curtis and Pennington [2], which covered only small changes of gauge. This weak gauge dependence is in marked contrast to the rainbow approximation, for which the critical coupling changes by 60% between just the Landau and Feynman gauges [2].

2. Curtis-Pennington equations

In Table 1 we have summarized the equations of the Curtis-Pennington ansatz. The photon propagator is taken bare – the quenched approximation – with covariant gauge parameter ξ . The fermion propagator has

Table 1
Curtis-Pennington Equations

$$\begin{aligned}
D_F^{\mu\nu}(k) &= \frac{1}{k^2} \left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2} \right) - \xi \frac{k^\mu k^\nu}{k^4} & S_F(p) &= \frac{\mathcal{Z}(-p^2)}{\gamma_\mu p^\mu - \mathcal{M}(-p^2)} \\
\Gamma^\mu(q, p) &= \frac{1}{2} \gamma^\mu \left[\frac{1}{\mathcal{Z}(-q^2)} + \frac{1}{\mathcal{Z}(-p^2)} \right] + \frac{1}{2} \left\{ \frac{(q+p)^\mu \gamma^\nu (q+p)_\nu}{q^2 - p^2} + \frac{(q^2 + p^2) [\gamma^\mu (q^2 - p^2) - (q+p)^\mu \gamma^\nu (q-p)_\nu]}{(q^2 - p^2)^2 + [\mathcal{M}^2(-q^2) + \mathcal{M}^2(-p^2)]^2} \right\} \left\{ \frac{1}{\mathcal{Z}(-q^2)} - \frac{1}{\mathcal{Z}(-p^2)} \right\} \\
&\quad - \frac{(q+p)^\mu}{q^2 - p^2} \left[\frac{\mathcal{M}(-q^2)}{\mathcal{Z}(-q^2)} - \frac{\mathcal{M}(-p^2)}{\mathcal{Z}(-p^2)} \right] \\
\mathcal{Z}^{-1}(x) &= 1 - \frac{\alpha}{4\pi} \int_0^{\Lambda^2} \frac{dy}{y + \mathcal{M}^2(y)} I(y, x) + \xi \frac{\alpha}{4\pi} \int_x^{\Lambda^2} \frac{dy}{y + \mathcal{M}^2(y)} \frac{\mathcal{Z}(y)}{\mathcal{Z}(x)} \\
\mathcal{Z}^{-1}(x) \mathcal{M}(x) &= \frac{\alpha}{4\pi} \int_0^{\Lambda^2} \frac{dy}{y + \mathcal{M}^2(y)} J(y, x) + \xi \frac{\alpha}{4\pi} \int_x^{\Lambda^2} \frac{dy}{y + \mathcal{M}^2(y)} \mathcal{M}(x) \frac{\mathcal{Z}(y)}{\mathcal{Z}(x)} \\
I(y, x) &= \frac{3}{2} \frac{1}{y-x} \left\{ \mathcal{M}(y) \left[\mathcal{M}(y) - \frac{\mathcal{Z}(y)}{\mathcal{Z}(x)} \mathcal{M}(x) \right] + \frac{1}{2} \frac{(y+x) [\mathcal{M}^2(y) + \mathcal{M}^2(x)]^2}{(y-x)^2 + [\mathcal{M}^2(y) + \mathcal{M}^2(x)]^2} \left[1 - \frac{\mathcal{Z}(y)}{\mathcal{Z}(x)} \right] \right\} \left[\frac{y^2}{x^2} \theta(x-y) + \theta(y-x) \right] \\
&\quad - \xi \left\{ 1 + \frac{\mathcal{M}(y)}{y} \left[\mathcal{M}(y) - \frac{\mathcal{Z}(y)}{\mathcal{Z}(x)} \mathcal{M}(x) \right] \right\} \frac{y^2}{x^2} \theta(x-y) \\
J(y, x) &= \frac{3}{2} \mathcal{M}(y) \left\{ 1 + \frac{\mathcal{Z}(y)}{\mathcal{Z}(x)} + \frac{y^2 - x^2}{(y-x)^2 + [\mathcal{M}^2(y) + \mathcal{M}^2(x)]^2} \left[1 - \frac{\mathcal{Z}(y)}{\mathcal{Z}(x)} \right] \right\} \left[\frac{y}{x} \theta(x-y) + \theta(y-x) \right] \\
&\quad - \frac{3x}{2} \frac{\mathcal{Z}(y)}{\mathcal{Z}(x)} \frac{\mathcal{M}(y) - \mathcal{M}(x)}{y-x} \left[\frac{y^2}{x^2} \theta(x-y) + \theta(y-x) \right] + \xi \frac{\mathcal{Z}(y)}{\mathcal{Z}(x)} \mathcal{M}(y) \frac{y}{x} \theta(x-y)
\end{aligned}$$

the most general possible form, involving the mass-function, $\mathcal{M}(-p^2)$, and the wave-function, $\mathcal{Z}(-p^2)$ (these correspond respectively to Curtis and Pennington's Σ and F).

The physical mass of the fermion is defined to be the lowest position at which the denominator function in the fermion propagator,

$$S_F(p) = \mathcal{Z}(-p^2) \frac{\gamma_\mu p^\mu + \mathcal{M}(-p^2)}{p^2 - \mathcal{M}^2(-p^2)},$$

has a zero, which is therefore a solution, m , of

$$m = \mathcal{M}(-m^2).$$

On physical grounds, this singularity should be on the real timelike axis of p^2 and should be gauge-independent; note that it would be a pole only if the photon were given a fictitious mass: with a massless photon, the singularity is a branch-point, the nature of which depends on the gauge. What Curtis and Pennington call the 'Euclidean mass', namely the lowest solution of

$$M = \mathcal{M}(M^2),$$

is not the same as the physical mass, m , and it is not expected to be exactly gauge-invariant. If one is going to abandon the attempt to calculate m , as one well might

do in view of the Atkinson-Blatt complex branch-points [3], one might perhaps take $\mathcal{M}(0)$ as an ersatz *effective mass*. At best one might hope it to be approximately gauge-invariant, on the grounds that it should be close to the physical mass m , which is gauge-invariant, at least in exact QED, or in a quenched approximation in which the first two Ward-Takahashi identities are respected [4].

The value of the wave-function at an arbitrarily selected renormalization point, μ , is defined to be the wave-function renormalization constant, which is conventionally dubbed Z_2 :

$$Z_2 = \mathcal{Z}(\mu^2).$$

It is convenient to choose the renormalization point to be Euclidean; the renormalized wave function is specified by

$$\tilde{\mathcal{Z}}(x) = Z_2^{-1} \mathcal{Z}(x). \quad (1)$$

The Curtis-Pennington ansatz defines a renormalizable scheme, so that in it $\tilde{\mathcal{Z}}(x)$ has a finite limit as the ultra-violet regularization is removed. The renormalized wave-function contains no explicit cut-off, but it is dependent on the renormalization point, and on the gauge parameter.

Chiral symmetry breaking occurs if the coupling, α , is greater than a certain critical value, α_c . This critical coupling is potentially a physically measurable quantity, since it signals a change of phase, and so it should be gauge invariant. Although this is not exactly true in the Curtis-Pennington system, it is approximately so. Indeed, the requirement that α_c be gauge-invariant could perhaps be used to specify further the form of the ansatz for the vertex function. In particular, the second term in the expression between the first set of parentheses $\{.. \}$ in the formula for Γ^μ in Table 1 is not uniquely determined, and the above requirement might with profit be used to refine this transverse part of the vertex.

The basic coupled integral equations are given in the third and fourth lines of Table 1, the complicated kernels I and J being explicit functions of \mathcal{M} and \mathcal{Z} . These equations were given in [2]¹.

3. Bifurcation equations

As can be seen from Table 1, the complete Curtis-Pennington equations are nonlinear and complicated. Clearly $\mathcal{M}(x) \equiv 0$ is always a possible solution; but it is not the one in which we are interested. However, the equations simplify at the critical point, where a nontrivial solution *bifurcates* away from the trivial one. To investigate this critical point, we have to take the Fréchet derivative of the nonlinear operators with respect to $\mathcal{M}(x)$ and evaluate it at the trivial ‘point’, $\mathcal{M}(x) \equiv 0$. This amounts in fact simply to throwing away all terms that are quadratic or higher in the mass function. It must be emphasized that this is *not* an approximation; it is a precise manner to locate the critical point by applying bifurcation theory.

Up to terms linear in $\mathcal{M}(x)$ and $\mathcal{M}(y)$, the kernels I and J reduce to

$$I(y, x) = -\xi \frac{y^2}{x^2} \theta(x - y) + \mathcal{O}(\mathcal{M}^2), \quad (2)$$

¹ They first appeared in [5] – note the misprints corrected in the erratum. In [5], the $\Lambda \rightarrow \infty$ limit was taken in a way that failed to respect axial current conservation, and so it was incorrectly deduced that chiral symmetry breaking occurs for all values of the coupling. This was rectified in [2].

$$\begin{aligned} J(y, x) = & \frac{3}{2} \mathcal{M}(y) \\ & \times \left\{ 1 + \frac{\mathcal{Z}(y)}{\mathcal{Z}(x)} + \frac{y+x}{y-x} \left[1 - \frac{\mathcal{Z}(y)}{\mathcal{Z}(x)} \right] \right\} \frac{y}{x_{>}} \\ & - \frac{3}{2} x \frac{\mathcal{Z}(y)}{\mathcal{Z}(x)} \frac{\mathcal{M}(y) - \mathcal{M}(x)}{y-x} \frac{y^2}{x_{>}^2} \\ & + \xi \frac{\mathcal{Z}(y)}{\mathcal{Z}(x)} \mathcal{M}(y) \frac{y}{x} \theta(x - y) + \mathcal{O}(\mathcal{M}^3), \end{aligned} \quad (3)$$

where $x_{>} = \max(x, y)$, and where it is enough to evaluate $\mathcal{Z}(x)$ to zeroth order in $\mathcal{M}(x)$, which can be done by inserting Eq. (2) into the equation for the wave-function, which accordingly becomes

$$\left[1 + \frac{\alpha\xi}{8\pi} \right] \mathcal{Z}(x) = 1 - \frac{\alpha\xi}{4\pi} \int_x^{\Lambda^2} \frac{dy}{y} \mathcal{Z}(y).$$

The unique solution of this is [1]

$$\mathcal{Z}(x) = \frac{1}{1 + \alpha\xi/8\pi} \left(\frac{x}{\Lambda^2} \right)^\nu, \quad (4)$$

where

$$\nu = \frac{2\alpha\xi}{8\pi + \alpha\xi}, \quad (5)$$

in agreement with lowest order perturbation theory.

On putting this solution for $\mathcal{Z}(x)$ into Eq. (3), we find the following first-order equation for $\mathcal{M}(x)$:

$$\begin{aligned} \left[1 + \frac{\alpha\xi}{8\pi} \right] \left(\frac{x}{\Lambda^2} \right)^{-\nu} \mathcal{M}(x) = & \frac{3\alpha}{8\pi} \int_0^{\Lambda^2} dy \mathcal{M}(y) \\ & \times \left\{ 1 + \left(\frac{y}{x} \right)^\nu + \frac{y+x}{y-x} \left[1 - \left(\frac{y}{x} \right)^\nu \right] \right\} \frac{1}{x_{>}} \\ & - \frac{3\alpha}{8\pi} \int_0^{\Lambda^2} dy \left(\frac{y}{x} \right)^\nu \frac{\mathcal{M}(y) - \mathcal{M}(x)}{y-x} \frac{xy}{x_{>}^2} \\ & + \frac{\alpha\xi}{4\pi} \int_0^x \frac{dy}{y} \left(\frac{y}{x} \right)^{\nu+1} \mathcal{M}(y) \\ & + \frac{\alpha\xi}{4\pi} \mathcal{M}(x) x^{-\nu} \int_x^{\Lambda^2} dy y^{\nu-1}. \end{aligned} \quad (6)$$

After evaluation of the last integral above, the term on the left-hand side cancels. This is a consequence

of the renormalizability of the Curtis-Pennington approximation. The ultra-violet cut-off can now be taken to infinity, and after canceling a factor of α throughout the equation, we find

$$\begin{aligned} \mathcal{M}(x) = & \frac{3\nu}{2\xi} \int_0^\infty \frac{dy}{x_{>}} \\ & \times \left\{ 1 + \left(\frac{y}{x}\right)^\nu + \frac{y+x}{y-x} \left[1 - \left(\frac{y}{x}\right)^\nu \right] \right\} \mathcal{M}(y) \\ & - \frac{3\nu}{2\xi} \int_0^\infty dy \frac{x_{<}}{x_{>}} \left(\frac{y}{x}\right)^\nu \frac{\mathcal{M}(y) - \mathcal{M}(x)}{y-x} \\ & + \nu \int_0^x \frac{dy}{y} \left(\frac{y}{x}\right)^{\nu+1} \mathcal{M}(y), \end{aligned} \quad (7)$$

with $x_{<} = \min(x, y)$. The last equation is scaling invariant, and it is solved by

$$\mathcal{M}(x) = x^{-s}, \quad (8)$$

on condition that s satisfies

$$\begin{aligned} f(\xi, s, \nu) \equiv & \xi - \frac{3\nu(\nu - s + 1)}{2(1 - s)} \\ & \times \left[3\pi \cot \pi(\nu - s) + 2\pi \cot \pi s - \pi \cot \pi \nu \right. \\ & \left. + \frac{1}{\nu} + \frac{1}{\nu + 1} + \frac{2}{1 - s} + \frac{3}{s - \nu} + \frac{1}{s - \nu - 1} \right] \\ = & 0 \end{aligned} \quad (9)$$

where the region of the $s - \nu$ plane for the convergence of the integral in Eq. (7) is specified by $s > 0$, $\nu < 1$, $s - \nu < 1$.

In a chosen gauge specified by ξ , this equation defines roots s for any value of the coupling α . Bifurcation occurs when two of these roots [with $s \in (0, 2)$] are equal. Then $\alpha \equiv \alpha_c$. To understand how and when this happens, it is easiest to consider first the situation in the Landau gauge, $\xi = 0$ i.e. $\nu = 0$. Then Eq. (9) is particularly simple and has just two roots in $(0, 2)$ for each value of α . For small α these roots are real. As α is increased, they approach one another, becoming equal at criticality, when $\alpha_c = 0.933667$. The boundary conditions imposed by Eq. (6) at $x = \Lambda^2$ demand that the behaviour of the mass function be oscillatory,

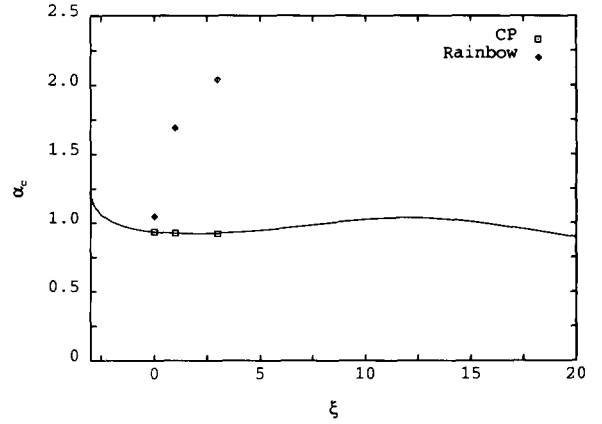


Fig. 1. Critical coupling, α_c , as a function of the gauge parameter, ξ (solid line). CP labels the numerical results of [2]. The corresponding values for the rainbow approximation are also from [2].

and that implies that the roots in Eq. (9) are complex. Thus only for α greater than α_c do Eq. (6) have a non-zero solution for $\mathcal{M}(x)$: only then can chiral symmetry breaking occur.

In other than the Landau gauge, particularly when ξ is large, Eq. (9) has more than two roots for s in $(0, 2)$, but we are of course interested in the ones that are continuously connected to the two that are present in the Landau gauge. A necessary condition for equality of two roots is $\partial f(\xi, s, \nu) / \partial s = 0$, i.e.

$$\begin{aligned} 2\pi^2 \csc^2 \pi s - 3\pi^2 \csc^2 \pi(\nu - s) + \frac{3}{(\nu - s)^2} \\ - \frac{2}{(1 - s)^2} + \frac{(1 - 2\xi/3)}{(1 + \nu - s)^2} = 0 \end{aligned} \quad (10)$$

Simultaneous solution of Eqs. (9), (10), (5) gives α_c as a function of ξ . This work cannot be performed analytically; but numerical procedures built into *Maple* and *Mathematica* have both been used to obtain $\alpha_c(\xi)$. Indeed the whole procedure can be automatized by using the *FindRoot* function of *Mathematica*.

The results are listed in Table 2 and illustrated in Fig. 1, where we have plotted the critical α_c against ξ over the rather large domain $-3 \leq \xi \leq 20$. The most important thing to note is the reassuringly weak gauge dependence. This is in keeping with the expectations from the results of [2] at $\xi = 0, 1$ and 3. That analysis involved the numerical solution of the fully coupled equations of Table 1 on a fine mesh of values of α , followed by an extrapolation $\mathcal{M}(x) \rightarrow 0$ to obtain

Table 2
Critical coupling, α_c , for a range of the gauge parameter, ξ

ξ	α_c
20	0.896171
18	0.945096
16	0.990483
15	1.009670
14	1.024887
12	1.038213
10	1.025070
8	0.993709
6	0.959386
5	0.944740
4	0.932991
3	0.924882
2	0.921272
1	0.923439
0	0.933667
-1	0.956804
-2	1.006606
-3	1.237124

the critical value α_c . Agreement with the present results (which of course can be easily obtained to many decimal places) is to several parts per mil: the results of [2] fall squarely on the curve, as can be seen in Fig. 1. At $\xi = 0$ we find $\alpha_c = 0.934$ and at $\xi = 1$ we have $\alpha_c = 0.923$. The curve has a local minimum at $\xi = 1.830$, where $\alpha_c = 0.921$. Thereafter α_c increases and at $\xi = 5$, for example, it has risen to 0.945. For larger values of the gauge parameter, the curve turns down again, the maximum occurring at $\xi_{\max} = 12.00$, with $\alpha_c(\xi_{\max}) = 1.038$. For ξ still larger, α_c decreases. It turns out that the solution of Eqs. (9), (10) always gives values for s , ν , for which the integral of Eq. (7) converges.

In contrast, for negative ξ , α_c increases and has an interesting cusp at $\xi = -3$. This is brought about by the cancellation of several terms on the right hand side of Eq. (6). Thus α_c rises steeply as $\xi \rightarrow -3$, with $\alpha_c(-3) = 1.237$. For $\xi < -3$, Eq. (7) is infra-red divergent. However, for $\alpha > \alpha_c$ this potential divergence is suppressed by terms quadratic in the mass-function: y is replaced by $y + \mathcal{M}^2(y)$ at crucial places in the denominators [6]. The solution then no longer has exactly the power form of Eq. (8), but asymptotically [$x \gg \mathcal{M}^2(x)$] this behaviour is still valid, and this is all we need to extend the bifurcation analysis.

In solving the bifurcation equation, we have at the

same time found the exponent s of Eq. (8). This too is only weakly gauge dependent in a sizeable region of ξ . For instance, in the Landau gauge ($\xi = 0$) $s = 0.4710$, while with $\xi = 5$, $s = 0.4551$. This exponent determines the ultra-violet behaviour of the mass-function $\mathcal{M}(-p^2)$ and is consequently related to γ_m , the anomalous dimension of the $\bar{\psi}\psi$ operator by $\gamma_m = 2(1 - s)$. Thus in the Landau gauge $\gamma_m = 1.058$, close to the value 1 that holds in the rainbow approximation and Holdom claims is exactly true in all gauges [7].

The fact that the variation of the critical coupling, over rather a large range of the gauge parameter, is only a couple of percent, which indicates the vast superiority [8] of the Curtis-Pennington ansatz over previous Ansätze for the vertex function that various people [9,10], including one of the present authors [11], have made in the past in an attempt to improve on the ladder approximation. Thus Rembiesa [9], using the gauge technique of Delbourgo, Salam and West [12], finds $\alpha_c = \pi/(3 + \xi)$. This dramatic gauge dependence can be traced to the fact that his fermion functions do not agree with perturbation theory in the weak coupling regime – a limit that should surely be respected to be physically relevant. In complete contrast, Kondo [13] finds a gauge-independent coupling, $\alpha_c = \pi/3$, but at the expense of using a vertex that has unintended singularities corresponding to the exchange of massless scalar particles – scalars not present in his original Lagrangian. The Curtis-Pennington vertex ansatz avoids these deficiencies.

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